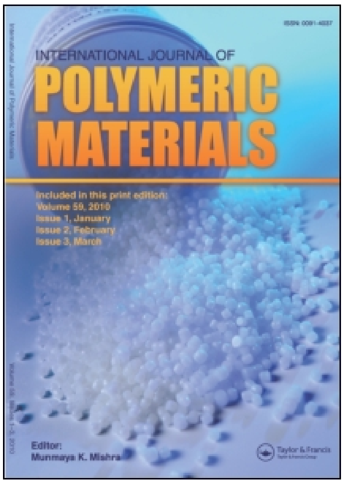

International Journal of Polymeric Materials
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713647664

Simulation of Linear Viscoelastic Behaviour using Simple Electrical Devices

S. Brüller†[a]
[a] Department of Mechanics, Technion-Israel Institute of Technology, Haifa, Israel

Intern. J. Polymeric Mater., 1976, Vol. 5, pp. 125–132

Printed in Reading, England

Simulation of Linear Viscoelastic Behaviour using Simple Electrical Devices

S. BRÜLLER†

Department of Mechanics, Technion—Israel Institute of Technology, Haifa, Israel 32000

(*Received November 4, 1974*)

By the use of very simple electrical equipment, the simulation of the response of linear viscoelastic materials to quasi-static loadings is made possible. Experiments carried out over extended periods with various loading programs of strain or stress may be simulated with good accuracy in periods as short as desired.

1 INTRODUCTION

The mechanical response of a linear viscoelastic material to quasi-static loading histories is usually obtained using numerical methods with the help of high speed computers. This procedure is generally laborious and expensive, and results are often calculated to a precision greater than warranted by the accuracy of observations. This paper presents a simple and practical method based on the analogy between the properties of linear viscoelastic materials and simple *RC* circuits. The electrical equipment used is very simple and inexpensive. In spite of this, the results obtained were in good agreement with the viscoelastic behaviour.

2 EQUATIONS OF VISCOELASTICITY

The time behaviour of a linear viscoelastic material can be described using the well-known relaxation modulus or creep compliance representation. If the

†Department of Mechanics, Technion—Israel Institute of Technology, Haifa, Israel 32000.

relaxation modulus is used, the stress-strain-time equation is:

$$\sigma(t) = \int_0^t E(t-\tau)\frac{\partial\epsilon(\tau)}{\partial\tau}\,d\tau \tag{1}$$

where: $\sigma(t)$ is the stress component
$E(t)$ is the relaxation modulus
$\epsilon(t)$ is the strain component.

A good approximation for the relaxation modulus uses a Prony–Dirichlet series of the form:

$$E(t) = E_0 + \sum_{i=1}^{n} E_i e^{-(t/\tau_i)} \tag{2}$$

If approximation (2) is used, Eq. (1) can be rewritten as an operator equation [2]:

$$\sigma = \left[E_0 + \sum_{i=1}^{n} \frac{E_i}{(1/\tau_i + d/dt)}(d/dt)\right]\epsilon \tag{3}$$

In the same manner, if the creep compliance $J(t)$ is chosen to represent the viscoelastic behaviour:

$$\epsilon(t) = \int_0^t J(t-\tau)\,\frac{\partial\sigma}{\partial\tau}\,d\tau \tag{4}$$

and $J(t)$ may also be approximated by:

$$J(t) = J_0 + \sum_{i=1}^{n} J_i(1 - e^{-(t/\tau_i)})$$

The following operator equation can be written [2]:

$$\epsilon = \left[J_0 + \sum_{i=1}^{n} \frac{J_i}{\tau_i(1/\tau_i + d/dt)}\right]\sigma \tag{6}$$

3 THE ELECTRIC-VISCOELASTIC ANALOGY

In order to recapitulate the behaviour of simple RC circuits with time, let us consider two typical circuits as in Figure 1. Here S represents a source, R a resistance and C a condenser:

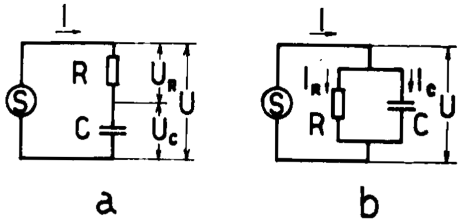

FIGURE 1 Simple RC circuits. a = Series. b = Parallel.

a) For the series circuit (Figure 1a):
The current I is (see for example [1])

$$I = C\frac{dU_C}{dt} = \frac{U_R}{R} \tag{7}$$

The total voltage is:

$$U = U_R + U_C \tag{8}$$

from which by differentiating with respect to time and using (7) we obtain:

$$\frac{dU}{dt} = \left(\frac{1}{C} + R\frac{d}{dt}\right) I \tag{9}$$

Eq. (9) can be written as an operator equation of the form:

$$I = \frac{1/R}{1/T + d/dt}\frac{dU}{dt} \tag{10}$$

where $T = RC$ is the time-constant of the circuit.

Connecting in parallel n such simple series circuits with an additional resistance R_0 (Figure 2), the following operator equation is obtained:

$$I = \left[\frac{1}{R} + \sum_{i=1}^{n} \frac{1/R_i}{(1/T_i + d/dt)}\left(\frac{d}{dt}\right)\right] U \tag{11}$$

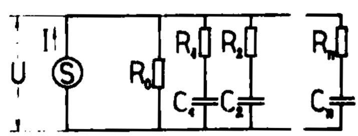

FIGURE 2 Parallel connected series circuits.

It is obvious that there exists a perfect analogy between Eqs. (3) and (11).

b) For the parallel circuit (Figure 1b):

The total current I is:

$$I = I_C + I_R = C(dU/dt + U/R) \tag{12}$$

Eq. (12) can be written as the operator equation:

$$U = \frac{R}{T(1/T + d/dt)} I \tag{13}$$

where $T = RC$ is the time-constant of the circuit.

Connecting in series n such simple parallel RC circuits with an additional resistance R_0 (Figure 3) the following operator equation is obtained:

$$U = \left[R + \sum_{i=1}^{n} \frac{R_i}{T_i\,(1/T_i + d/dt)} \right] I \tag{14}$$

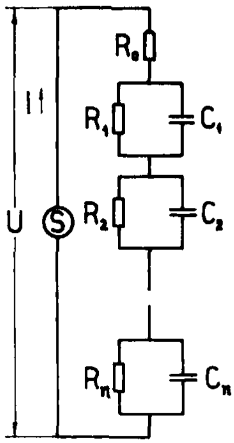

FIGURE 3 Series connected parallel circuits.

Eqs. (6) and (14) are perfectly analogous.

In conclusion, in the above analogies, the following replacements may be made:

$$\begin{array}{lll} \sigma & \text{by} & I \\ \epsilon & \text{by} & U \\ E_i & \text{by} & 1/R_i \\ J_i & \text{by} & R_i \end{array} \tag{15}$$

4 THE SCALE FACTORS

Comparing the discrete distributions of the appropriate viscoelastic relaxation modulus or creep compliance on the one hand and the response of the electrical

RC circuits on the other, two scale factors must be taken into account. These are:

a) *The time scale-factor* α *given by*:

$$\alpha = \frac{\tau_i}{T_i} \; (i = 1, 2, \ldots, n) \tag{16}$$

In the case of interest, $\alpha > 1$, the extended viscoelastic experiment will be described by a brief electrical experiment, α times shorter in duration.

b) *The magnitude scale-factor* β *given by*:

$$\beta = \frac{\sigma}{I} \qquad \text{or} \qquad \beta = \frac{\epsilon}{U} \tag{17}$$

Different combinations of R and C can be used to obtain the time constant, and the most suitable combination will be selected without previous concern for the value of β. This will be calculated from the experimental data as shown later.

5 EXAMPLES

a) Let us take, as a first example, published data for a relaxation experiment of a rubber compound commercially known as Perbunan P 21 [3].

At 23°C, a very good approximation of the relaxation modulus is given by:

$$E(t) = 95.2 + 2.79e^{-(t/2273)} + 2.19e^{-(t/348)} + 2.50e^{-(t/62)} \tag{a}$$

in which $E(t)$ is given in kg/cm^2 and t in sec. Data were given over an experimental period of 80 min.

We will select $\alpha = 100$ in order to reproduce the viscoelastic experiment in 0.8 min = 48 sec. According to Eq. (16) the time-constants of the three series circuits will be:

$$\begin{aligned} T_1 &= 22.73 \text{ sec} \\ T_2 &= 3.48 \text{ sec} \\ T_3 &= 0.62 \text{ sec.} \end{aligned} \tag{b}$$

Using the electric-viscoelastic analogy, the values of the resistances are proportional to the reciprocals of the coefficients of Eq. (a), i.e.:

$$\begin{aligned} R_0 &\rightarrow 1/95.2 = 1.05 \times 10^{-2} \\ R_1 &\rightarrow 1/27.9 = 3.58 \times 10^{-1} \\ R_2 &\rightarrow 1/2.19 = 4.56 \times 10^{-1} \\ R_3 &\rightarrow 1/2.5 = 4.00 \times 10^{-1} \end{aligned} \tag{c}$$

These can be multiplied by any constant desired, and for our purposes, the

J

most suitable values would seem to be:

$$\begin{aligned} R_0 &= 0.105\ \mathrm{M\Omega} \\ R_1 &= 3.58\ \ \mathrm{M\Omega} \\ R_2 &= 4.56\ \ \mathrm{M\Omega} \\ R_3 &= 4.00\ \ \mathrm{M\Omega} \end{aligned} \tag{d}$$

From $T_i = R_i C_i$ the capacitances to be used are:

$$\begin{aligned} C_1 &= 6.35\ \ \mu\mathrm{F} \\ C_2 &= 0.764\ \mu\mathrm{F} \\ C_3 &= 0.155\ \mu\mathrm{F} \end{aligned} \tag{e}$$

The magnitude scale-factor will be calculated assuming for an elongation of 25% ($\epsilon = 0.25$), a corresponding voltage $U = 25\mathrm{V}$.

At $t \to \infty$ the stress will be 23.8 kg/cm^2 while the current is 0.238 mA. Hence

$$\beta = 0.01\ \frac{1}{V} \quad \text{or} \quad \frac{\mathrm{mA}}{\mathrm{kg/cm^2}} \tag{f}$$

The four resistances and the three condensers are connected in a scheme shown in Figure 2 with a voltmeter parallel to the source and a recorder for the current. The recorder will then trace the simulated "stress" (current) output obtained from any "strain" (voltage) input history.

To reproduce the relaxation experiment as presented by Raous [3], a constant voltage $U = 25\mathrm{V}$ representing a 25% elongation is to be suddenly applied to the device and maintained during 48 sec. The simulated relaxation curve is given in Figure 4. The solid line agrees within the limit of graphical accuracy with the experimental curve [3]. The response system of the recorder causes the upsurge which has been replaced by the dotted line.

b) As a second example, the creep behaviour of a specimen of unplasticized P.V.C. plastic in a test [5] in which there were five separate loading steps, will be simulated. Using a method as presented in [4], the time dependence of the stress during the first loading step has been found to be described by the creep compliance:

$$J(t) = 10^{-6}[2.34 + 0.134(1 - e^{-(t/15)}) + 0.060(1 - e^{-(t/120)})] \tag{g}$$

$J(t)$ is given in in^2/lb and t in min.

Assuming $\alpha = 50$ we have:

$$\begin{aligned} T_1 &= 18\ \mathrm{sec} \\ T_2 &= 144\ \mathrm{sec} \end{aligned} \tag{h}$$

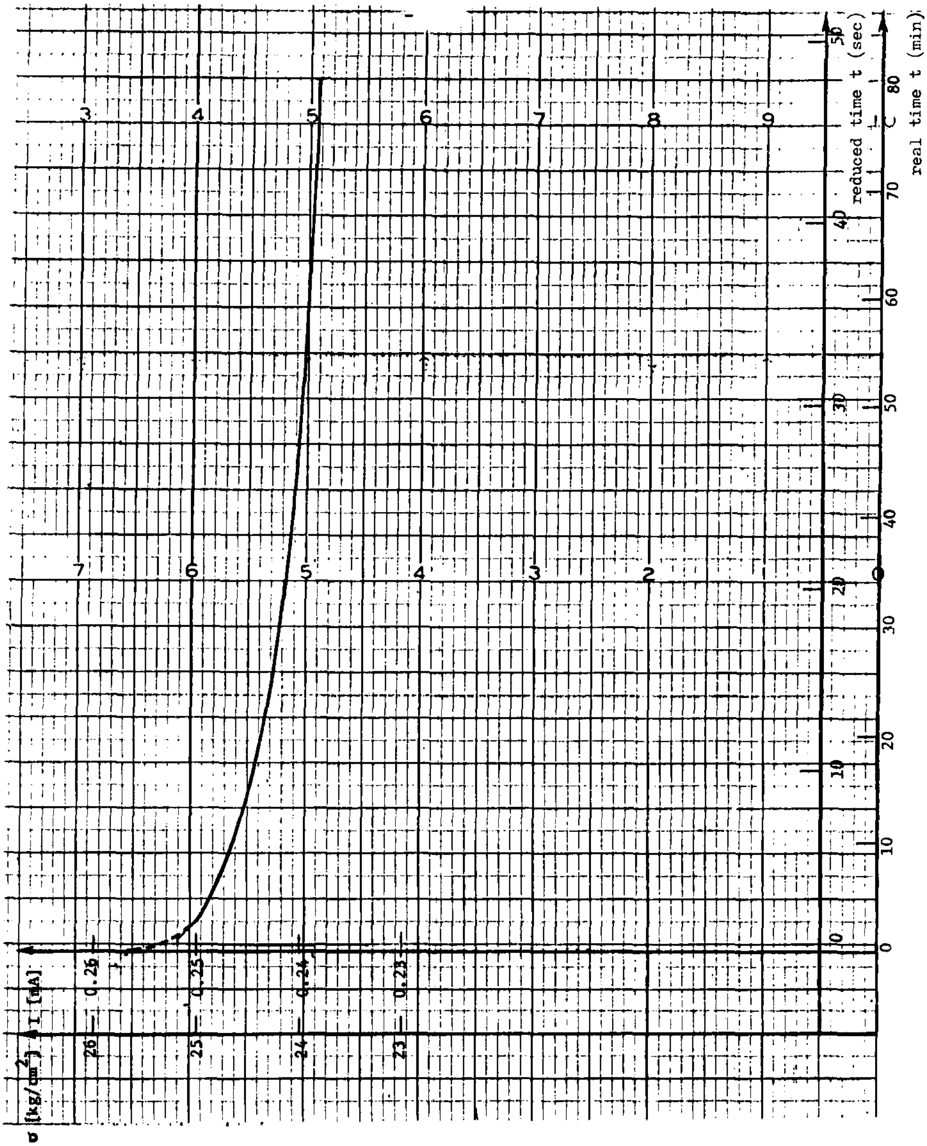

FIGURE 4 The simulated relaxation curve of example (a).

We choose the resistances:

$$\begin{aligned} R_0 &= 23.4\ \mathrm{M}\Omega \\ R_1 &= 1.34\ \mathrm{M}\Omega \\ R_2 &= 0.60\ \mathrm{M}\Omega \end{aligned} \quad \text{(i)}$$

The capacitances are:

$$\begin{aligned} C_1 &= 13.4\ \mu\mathrm{F} \\ C_2 &= 240\ \mu\mathrm{F} \end{aligned} \quad \text{(j)}$$

The three resistances and two condensers should be connected in a scheme as shown in Figure 3. A recorder connected as voltmeter will note the corresponding "strain" (voltage) output while the input is obtained from a constant current source step-wise externally activated in accordance with the loading program in the reduced time scale.

6 CONCLUSIONS

The simulation of stress-strain-time relations of viscoelastic materials for any desired loading history is possible using simple electrical devices. The response can be directly recorded. Only very simple calculations are needed.

Very good agreement between the real and the simulated response is obtained.

References

1. R. J. Smith, *Circuits, Devices and Systems*, John Wiley & Sons Inc. (1967).
2. M. L. Williams, Structural analysis of viscoelastic materials, *AIAA Journal* **2**, 5, 785–808 (May 1964).
3. M. Raous, Determination du module d'young operationel d'un corps viscoelastique a partir de sa courbe de relaxation, *Communication presentée au VIe Congrès International de Rhéologie*. Lyon (Sept. 1972).
4. A. Foux and S. Brüller, The linear viscoelastic range of perspex and epoxy resin, *Isr. J. of Techn.* **9**, 5, 507–514 (Sept. 1971).
5. A. C. Pipkin and T. G. Rogers, A non-linear integral representation for viscoelastic behaviour, *J. Mech. Phys. Solids*, **16**, 59–72 (1968).